International Journal of Theoretical Physics, Vol. 38, No. 4, 1999

Adiabatic Interpretation of Particle Creation in a de Sitter Universe

C. Molina-París[1]

Received September 15, 1998

The choice of vacuum state for a quantum scalar field (massive and arbitrarily coupled to the gravitational field, with coupling constant ξ) propagating in a de Sitter spacetime is discussed. The problem of finite-time initial conditions for the mode functions is analyzed, as well as how these determine the vacuum state of the quantum system. The principle guiding the choice of vacuum state is the following: one wants the vacuum contribution to the energy-momentum tensor to contain all the ultraviolet divergent terms, so that the particle creation terms are finite, and covariantly conserved. There is a suitable set of modes (*instantaneous adiabatic basis*) in which this splitting of the expectation value of the energy-momentum tensor can be carried out. Numerical results are presented for different initial times and the following values for the mass and the coupling constant: $m = 0.6$, $\xi = 1/6$. The nature of the particle creation effect is described and its relationship to the concept of a horizon crossing time is shown. These numerical results imply that back reaction can be important and should be the subject of further research.

1. INTRODUCTION

The theory of quantum fields in a curved (classical and fixed) gravitational background does not provide a general principle that selects a natural physical vacuum state. In the absence of symmetries (in the underlying spacetime) the choice of a vacuum state for the quantum field becomes a difficult task.[2] In this paper we study a quantum scalar field, not conformally invariant (massless and $\xi = 1/6$) and choose an adiabatic vacuum as the initial state for the quantum field, which is general enough and does not

[1] Theoretical Division T-6 and T-8, Los Alamos National Laboratory, Los Alamos, New Mexico 87545; e-mail: carmen@t6-serv.lanl.gov.

[2] In Minkowski spacetime the presence of the Poincaré group is the guiding principle, as it singles out a natural coordinate system with respect to which one defines a vacuum state.

0020-7748/99/0400-127 3$16.00/0

necessarily share all the FRW symmetries of the background geometry. The importance of these two choices is as follows:

1. It is known that particle creation in a conformally flat spacetime (i.e., FRW universe) requires a nonconformal field. This means that the quantum scalar field must be either massive or not conformally coupled to the curvature $\xi \neq 1/6$.
2. As the underlying theory does not choose a physical state, one needs to understand the physics of particle creation and its relationship to the choice of finite-time initial conditions.

The paper is organized as follows. In Section 2 an example is presented that makes explicit the importance of the choice of a physical vacuum state. In Section 3 the basic elements of quantum field theory in a curved background are introduced. Section 4 deals with the particular case of a de Sitter background. In Section 5 the principle guiding the choice of an adiabatic vacuum state is stated and its relationship with finite-time initial conditions is explained. Section 6 introduces the energy-momentum tensor of the system under consideration. In Section 7 the method of adiabatic regularisation is described. Section 8 introduces the concept of a generalized Bogoliubov transformation and its relationship with the concept of adiabatic particle. The renormalized energy-momentum tensor is calculated in this section. The numerical results for different initial conditions are shown in Section 9. The last section contains a brief summary of the results presented and further applications of the approach considered here.

2. CHARGED SCALAR QED IN MINKOWSKI SPACETIME

This example illustrates the importance of choosing a physical vacuum state [1]. Let us consider a charged scalar field Φ in Minkowski spacetime in the presence of a homogeneous electric field $\mathbf{E} = E\mathbf{z}$, and let us choose the vector potential as $\mathbf{A}(t) = -Et\mathbf{z}$. The wave equation for this field is given by

$$[(\partial_\mu - ieA_\mu)^2 + m^2]\Phi(t, \mathbf{x}) = 0. \tag{2.1}$$

The well-known symmetries of Minkowski spacetime make the previous equation separable (in Cartesian coordinates) and one can find solutions of the form (Fourier mode decomposition)[3].

[3] V is the volume of spacetime we are considering and in which we are quantizing the system.

$$\Phi(t, \mathbf{x}) = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} a_{\mathbf{k}} f_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + b_{\mathbf{k}}^{\dagger} f_{-\mathbf{k}}^{*}(t)\, e^{-i\mathbf{k}\cdot\mathbf{x}} \tag{2.2}$$

with mode functions that satisfy the following harmonic differential equation:

$$\ddot{f}_{\mathbf{k}}(t) + \omega_{\mathbf{k}}^2(t) f(t) = 0, \qquad \omega_{\mathbf{k}}^2\,(t) = (k_z + eEt)^2 + k_{\perp}^2 + m^2, \quad k_{\perp}^2 = k_x^2 + k_y^2. \tag{2.3}$$

We choose for the initial quantum state of the system the vacuum corresponding to this mode decomposition, that is, the state that is annihilated by all the $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$. These mode functions are invariant under time reversal, that is, $(t \to -t, \mathbf{k} \to -\mathbf{k})$, whereas the expectation value of the current (with respect to the vacuum state defined above) $\mathbf{j} = j_z \mathbf{z}$ is odd under this exchange. This is easy to see; suppose we consider solutions such that $|f_{\mathbf{k}}(t)| = |f_{-\mathbf{k}}(-t)|$; then we have the following expression for $\langle j_z \rangle$:

$$\langle j_z \rangle = \frac{2e}{V} \sum_{\mathbf{k}} (k_z + eEt) |f_{\mathbf{k}}(t)|^2. \tag{2.4}$$

Therefore in the vacuum state defined by $a_{\mathbf{k}}|0\rangle = 0 = b_{\mathbf{k}}|0\rangle$, the expectation value of the current vanishes. On the other hand, we know that there are solutions of equation (2.3) with adiabatic asymptotic behavior, such that

$$\lim_{t\to\pm\infty} f_{\mathbf{k}(\pm)}(t) = \tilde{f}_{\mathbf{k}}(t), \qquad \tilde{f}_{\mathbf{k}}(t) = \frac{1}{[2\omega_{\mathbf{k}}(t)]^{1/2}} \exp\left[-i \int^{t} dt'\, \omega_{\mathbf{k}}(t') \right] \tag{2.5}$$

and with $f_{\mathbf{k}(-)}(t) \neq f_{\mathbf{k}(+)}(t)$. Furthermore, these two families of solutions $\{f_{\mathbf{k}(-)}, f_{\mathbf{k}(-)}^{*}\}$ and $\{f_{\mathbf{k}(+)}, f_{\mathbf{k}(+)}^{*}\}$ are related by a Bogoliubov transformation and each of them represents a different vacuum state [2],

$$f_{\mathbf{k}(-)}(t) = \alpha_{\mathbf{k}} f_{\mathbf{k}(+)}(t) + \beta_{\mathbf{k}} f_{\mathbf{k}(+)}^{*}(t). \tag{2.6}$$

If the initial vacuum state (in the Heisenberg representation) is $|0_{(-)}\rangle$, that is, the adiabatic vacuum at very early times, it is easy to show that at later times, in the remote future, when the natural choice for a set of adiabatic observers is $\{f_{\mathbf{k}(+)}, f_{\mathbf{k}(+)}^{*}\}$, these *inertial observers* would detect particles. In particular the number of particles detected in the kth mode is given by the expectation value of the number operator $N_{\mathbf{k}(+)}$ (in the remote future basis) in the adiabatic vacuum at very early times, that is,[4]

[4] The same calculation can be performed to obtain the number of antiparticles detected in the adiabatic vacuum at very early times. One needs to replace $N_{k(+)}^{a}$ by $N_{k(+)}^{b}$.

$$\langle 0_{(-)}|N^{a}_{\mathbf{k}(+)}|0_{(-)}\rangle \stackrel{\text{def}}{=} \langle 0_{(-)}|a^{\dagger}_{\mathbf{k}(+)}a_{\mathbf{k}(+)}|0_{(-)}\rangle = |\beta^{2}_{\mathbf{k}}| \\ = \exp\left(\frac{-\pi k_{\perp}^{2} + m^{2}}{eE}\right) \neq 0. \tag{2.7}$$

Notice that the number of particles produced does not depend on k_z and it is time independent. These observers will measure a non-vanishing $\langle j_z\rangle$. This is the Schwinger effect, which would have been completely missed in the vacuum state that is time-reversal invariant. This example is very similar to a quantum scalar field in a de Sitter background, and their analogies will be made explicit in Section 4.

3. BASIC ELEMENTS OF QUANTUM FIELD THEORY IN A CURVED SPACETIME

This section is intended to give the basic elements that come to play when one is studying quantum fields in a classical gravitational background. The underlying theory is semiclassical quantum field theory in the sense that the matter fields are treated as quantum in nature, whereas the gravitational field is considered fixed and classical.

1. We need to specify the quantum state of the field Φ. Let us label it by $|\psi\rangle$, and assume that it is a pure quantum state. The fundamental equation governing the evolution of the system is the semiclassical Einstein equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} \stackrel{\text{def}}{=} R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle \tag{3.1}$$

2. The gravitational background is treated as a classical field and it is given *a priori*. This means that the left-hand side of the previous equation is totally determined by the background metric $g_{\mu\nu}$ of a Friedmann–Robertson–Walker cosmology.
3. It can be shown that $\langle\psi|T_{\mu\nu}|\psi\rangle$ expressed in a certain coordinate system yields the expectation values of the energy, pressure, and trace of the quantum field in the chosen state $|\psi\rangle$. One needs to develop a scheme to derive from these quantities the physical, renormalized and covariantly conserved ones.
4. The scheme used to regularize and renormalize the divergences of $\langle\psi|T_{\mu\nu}|\psi\rangle$ is adiabatic subtraction and the mode integrals will be calculated with a momentum cutoff [3].

5. One defines $\langle\psi|T_{\mu\nu}|\psi\rangle_R \stackrel{\text{def}}{=} \langle\psi|T_{\mu\nu}|\psi\rangle - \langle\psi|T_{\mu\nu}|\psi\rangle_A$ and by means of an adiabatic particle interpretation, one obtains physically intuitive expressions for the energy, pressure, and trace of the quantum field in the sense that they split naturally into a vacuum polarization term and a particle creation contribution.

4. QUANTUM SCALAR FIELD IN A DE SITTER SPACETIME

In this section we consider a quantum scalar field in a de Sitter gravitational background. We use a coordinate system [4] in which the spatial sections have curvature $\kappa = +1$, and the scale factor is $a(t) = Z^{-1}\cosh(Zt)$. In comoving coordinates the metric is given by

$$ds^2 = -dt^2 + C(t)h_{ij}(\mathbf{x})\, dx^i \otimes dx^j, \qquad g_{tt} = -1, \quad g_{ij} = C(t)h_{ij}(\mathbf{x}) \tag{4.1}$$

where $h_{ij}(\mathbf{x})$ is the metric of a three-dimensional sphere, and $C(t) = a^2(t)$. Notice that the scalar curvature of this four-dimensional manifold is $R = 12Z^2$.

The wave equation for a massive scalar field and arbitrarily coupled to the curvature is

$$[-\Box + m^2 + \xi R]\Phi(t, \mathbf{x}) = 0.$$

In the coordinate system we are using (comoving coordinates) the $\Box$ operator is given by

$$\Box f = -\ddot{f} - 3H\dot{f} + \frac{1}{C}\Delta_{(3)} f, \qquad H(t) = \frac{\dot{a}(t)}{a(t)} \tag{4.2}$$

and $\Delta_{(3)}$ is the Laplace–Beltrami operator on the three-dimensional spacelike hypersurfaces (spheres, in this case, as $\kappa = +1$). The equation of motion for the scalar field propagating in this cosmological model is

$$\Phi(t, \mathbf{x}) + 3H(t)\Phi(t, \mathbf{x}) - \frac{1}{C(t)}\Delta_{(3)}\Phi(t, \mathbf{x}) + [m^2 + \xi R(t)]\Phi(t, \mathbf{x}) = 0. \tag{4.3}$$

Due to the symmetry of the cosmological background this equation is separable and we can decompose the field Φ in terms of creation and annihilation operators as follows:

$$\Phi(t, \mathbf{x}) = \frac{1}{[a(t)]^{3/2}} \sum_{k,l,m} [a_{klm} f_k(t)\mathcal{Y}_{klm}(\mathbf{x}) + a^{\dagger}_{klm} f^*_k(t)\mathcal{Y}^*_{klm}(\mathbf{x})] \tag{4.4}$$

with

$$\Delta_{(3)}\mathcal{Y}_{klm}(\mathbf{x}) = -k(k+2)\mathcal{Y}_{klm}(\mathbf{x}), \tag{4.5}$$

$$k = 0,1,2,\ldots,\quad l = 0,1,2,\ldots,k-1,\quad m = -l,\ldots,+l.$$

The equation of motion for the mode functions $f_k(t)$ is given by

$$\ddot{f}_k(t) + \Omega_k^2(t)f_k(t) = 0 \tag{4.6}$$

with

$$\Omega_k^2(t) = Z^2\left[\gamma^2 + \left(k+\frac{1}{2}\right)\left(k+\frac{3}{2}\right)\operatorname{sech}^2(Zt)\right],$$

$$\gamma^2 = \frac{m^2}{Z^2} + 12\left(\xi - \frac{1}{6}\right) - \frac{1}{4}$$

In the remote past and future ($t \to \pm\infty$), Ω_k^2 tends to the constant (time independent) value $\lim_{t\to\pm\infty}\Omega_k^2 = Z^2\gamma^2$, and we can find exact solutions for the mode functions such that at early and late times, respectively, they tend to adiabatic mode functions. They are defined as follows:

$$\lim_{t\to\pm\infty} f_{k(\pm)}(t) = \tilde{f}_k(t),\qquad \tilde{f}_k(t) = \frac{1}{[2\Omega_k(t)]^{1/2}}\exp\left[-i\int^t dt'\Omega_k(t')\right]. \tag{4.7}$$

Notice that $f_{k(-)}(t) \neq f_{k(+)}(t)$ and that their corresponding vacuum states are nonunitarily equivalent. In fact these two families of solutions $\{f_{k(-)}, f^*_{k(-)}\}$ and $\{f_{k(+)}, f^*_{k(+)}\}$ are related by the Bogoliubov transformation [2]

$$f_{k(-)}(t) = \hat{\alpha}_k f_{k(+)}(t) + \hat{\beta}_k f^*_{k(+)}(t). \tag{4.8}$$

If the initial vacuum state for the quantum field (in the Heisenberg representation) is $|0_{(-)}\rangle$, that is, the adiabatic vacuum at very early times, it is easy to show that at later times, in the remote future, when the natural choice for a set of adiabatic observers is $\{f_{k(+)}, f^*_{k(+)}\}$, these *inertial observers* would detect particle production given by the expectation value of the number operator $N_{k(+)}$, (in the remote future basis) in the adiabatic vacuum at very early times, that is,

$$\langle 0_{(-)}|N_{k(+)}|0_{(-)}\rangle \stackrel{\text{def}}{=} \langle 0_{(-)}|a^\dagger_{klm(+)}a_{klm(+)}|0_{(-)}\rangle = |\hat{\beta}_k^2| = \operatorname{cosech}^2(\pi\gamma) \neq 0. \tag{4.9}$$

Notice that the number of particles produced does not depend on k and is time independent. These observers (adiabatic in the remote future) will measure a nonzero number of particles in the vacuum state $|0_{(-)}\rangle$. This is the Schwinger effect, which would have been completely missed in the vacuum state that is de Sitter invariant [5]. Notice the similarity between this section and Section 2 as mentioned there.

5. INITIAL CONDITIONS AND ADIABATIC VACUUM

The application of these results to the early universe requires finite-time initial conditions.[5] In order to be able to do a *particle interpretation* (within the limits that this concept has for an arbitrary spacetime), we introduce an adiabatic basis defined as follows:

$$\tilde{f}_k(t) \stackrel{\text{def}}{=} \frac{1}{[2W_k(t)]^{1/2}} \exp\left[-i \int_{t_0}^{t} dt' \, W_k(t') \right] \stackrel{\text{def}}{=} \frac{1}{[2W_k(t)]^{1/2}} e^{-iS_K(t)}$$

$$S_k(t) \stackrel{\text{def}}{=} \int_{t_0}^{t} dt' \, W_k(t'). \tag{5.1}$$

We want to allow for the most general solution of equation (4.6) that satisfies adiabatic initial conditions at time t_0. This implies that the exact solution f_k is related to the instantaneous adiabatic basis $\{\tilde{f}_k, \tilde{f}_k^*\}$ by a generalized Bogoliubov transformation

$$f_k(t) \stackrel{\text{def}}{=} \alpha_k(t)\tilde{f}_k(t) + \beta_k(t)\tilde{f}_k^*(t), \qquad |\alpha_k(t)|^2 - |\beta_k(t)|^2 = 1 \tag{5.2}$$

as both sets of modes satisfy the Wronskian conditions

$$f_k(t)\dot{f}_k^*(t) - \dot{f}_k(t)f_k^*(t), \qquad \tilde{f}_k(t)\dot{\tilde{f}}_k^*(t) - \dot{\tilde{f}}_k(t)\tilde{f}_k^*(t) = i. \tag{5.3}$$

Notice that this Bogoliubov transformation is a time-dependent one, that is, the coefficients α_k and β_k depend on time. We must also impose a condition on $\dot{f}_k(t)$ in order to uniquely determine these coefficients. It is important to mention at this point that f_k and $\dot{f}_k$ are independent, but they are constrained to satisfy the Wronskian condition. Therefore $\dot{f}_k$ has to be chosen in such a way as to preserve the condition on the Wronskian. The most general choice is

$$\dot{f}_k(t) \stackrel{\text{def}}{=} \left[-iW_k(t) + \frac{V_k(t)}{2} \right] \alpha_k(t)\tilde{f}_k(t) + \left[iW_k(t) + \frac{V_k(t)}{2} \right] \beta_k(t)\tilde{f}_k^*(t). \tag{5.4}$$

[5] One has in mind possible applications to the inflationary regime of the early universe.

For now we leave W_k and V_k without specification and only mention that both quantities are restricted to be real, and that at early and late times, W_k has to tend to Ω_k, and V_k has to vanish, so that we recover the "in" and "out" exact solutions in the "in" (remote past) and "out" (remote future) regions. Later, the problem at hand will guide us to decide which particular choice is more convenient. These two conditions imply

$$\alpha_k(t) = i f_k^*(t)\left[\dot{f}_k(t) - \left(iW_k(t) + \frac{V_k(t)}{2}\right) f_k(t)\right]$$
$$\beta_k(t) = -i f_k(t)\left[\dot{f}_k(t) + \left(iW_k(t) - \frac{V_k(t)}{2}\right) f_k(t)\right]. \qquad (5.5)$$

In order to consider the most general initial conditions, we assume the following:

1. At time t_0 the system is in the vacuum state $|0\rangle$ defined by the mode decomposition of equation (4.4). As we are working in the Heisenberg representation, this means that at any time, the system is in this vacuum state. Notice that this choice implies that the state considered represents a distribution of matter that is spatially homogeneous and isotropic, in consistency with the symmetries of the underlying gravitational background.
2. At time t_0 the initial value problem that we are going to analyze is the following:

$$f_k(t_0) \stackrel{\text{def}}{=} [2W_k(t_0)]^{-1/2}, \qquad \dot{f}_k(t_0) \stackrel{\text{def}}{=} \left[-iW_k(t_0) + \frac{V_k(t_0)}{2}\right] f_k(t_0). \quad (5.6)$$

   This condition restricts V_k to be a real function that vanishes at early and late times, and W_k to tend to Ω_k at late and early times, too, so that we recover the "in" and "out" asymptotic behavior

$$\lim_{t_0 \to \pm\infty} f_k(t_0) = f_{k(\pm)}(t_0). \qquad (5.7)$$

3. We define the particle occupation number in the kth mode, $\mathcal{N}_k(t)$, as follows:

$$\mathcal{N}_k(t) \stackrel{\text{def}}{=} \langle 0|\tilde{a}^\dagger_{klm}(t)\tilde{a}_{klm}(t)|0\rangle = |\beta_k(t)|^2. \qquad (5.8)$$

4. The adiabatic particle interpretation comes from the particular choice of initial conditions $\alpha_k(t_0) = 1$ and $\beta_k(t_0) = 0$; therefore, $\mathcal{N}_k(t_0) =$

0; that is, the initial state is an adiabatic vacuum state with respect to the instantaneous adiabatic basis $\{\tilde{f}_k, \tilde{f}_k^*\}$. In general $\mathcal{N}_k(t) \neq 0$ for $t > t_0$, and there will be particle creation in this adiabatic sense.

6. ENERGY-MOMENTUM TENSOR FOR A DE SITTER COSMOLOGY

The classical energy momentum tensor of a free scalar field can be written as

$$T_{\mu\nu} = (1 - 2\xi)(\partial_\mu \Phi)(\partial_\nu \Phi) + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu} g^{\alpha\beta}(\partial_\alpha \Phi)(\partial_\beta \Phi)$$
$$- 2\xi \Phi \nabla_\mu \nabla_\nu \Phi + 2\xi g_{\mu\nu} \Phi \Box \Phi + \xi\, G_{\mu\nu} \Phi^2 - \frac{1}{2} m^2 g_{\mu\nu} \Phi^2$$

and the trace is given by

$$T \overset{\text{def}}{=} g^{\mu\nu} T_{\mu\nu} = (6\xi - 1) g^{\mu\nu}(\partial_\mu \Phi)(\partial_\nu \Phi)$$
$$+ 6\xi \Phi \Box \Phi - \xi R \Phi^2 - 2m^2 \Phi^2. \tag{6.1}$$

If we make use of equation (4.2) we can write for the components of the energy-momentum tensor

$$T_{tt} \overset{\text{def}}{=} \epsilon = \frac{1}{2} \Phi\Phi - \frac{1}{2C} \Phi \Delta_{(3)} \Phi + 6\xi H \Phi\Phi$$
$$+ 3\xi \left(H^2 + \frac{\kappa}{C} \right) \Phi^2 + \frac{m^2}{2} \Phi^2 \tag{6.2}$$

$$g^{ij} T_{ij} \overset{\text{def}}{=} 3p = \left(\frac{3}{2} - 6\xi \right) \Phi\Phi + \frac{1}{2C} \Phi \Delta_{(3)} \Phi - 6\xi \phi \Phi - 12\xi H \Phi\Phi$$
$$- \xi \left(6\dot{H} + 9H^2 + \frac{3\kappa}{C} \right) \Phi^2 - \frac{3m^2}{2} \Phi^2 \tag{6.3}$$

$$T = (1 - 6\xi)\Phi\Phi + \frac{1}{C} \Phi \Delta_{(3)} \Phi - 6\xi \Phi\Phi - 18\xi H \Phi\Phi$$
$$- \xi R \Phi^2 - 2m^2 \Phi^2. \tag{6.4}$$

We know that the energy, the isotropic pressure and the trace are not independent variables, but are indeed related by the relation $T = -\epsilon + 3p$ (which is always a useful way to check one's calculation). If we make use of equation

(4.3) to get rid of the terms proportional to $\Phi\Phi$, we can write for the components of the classical $T_{\mu\nu}$,[6]

$$\epsilon = \frac{1}{2}\Phi\Phi - \frac{1}{2C}\Phi\Delta_{(3)}\Phi + 6\xi H\Phi\Phi + 3\xi\left(H^2 + \frac{\kappa}{C}\right)\Phi^2 + \frac{m^2}{2}\Phi^2 \tag{6.5}$$

$$T = (1 - 6\xi)\Phi\Phi + (1 - 6\xi)\frac{1}{C}\Phi\Delta_{(3)}\Phi + \xi(6\xi - 1)R\Phi^2 + (6\xi - 2)m^2\Phi^2. \tag{6.6}$$

We are now ready to calculate the expectation value of the energy-momentum tensor in the adiabatic vacuum state defined in the previous section. It is at this stage that we restrict our study to the case of $\xi = 1/6$, that is, a conformal coupling between the scalar field and the scalar curvature. It can be shown that it is no loss of generality [6]. The bare energy and trace are

$$\langle\epsilon\rangle = \frac{1}{2\pi^2 a^3}\sum_{k=0}^{+\infty}(k+1)^2\left[\frac{1}{2}|\dot{f}_k|^2 - \frac{H}{2}\Re(f_k^*\dot{f}_k) + \left(\frac{\omega_k^2}{2} + \frac{H^2}{8}\right)|f_k|^2\right] \tag{6.7}$$

$$\langle T\rangle = -\frac{m^2}{2\pi^2 a^3}\sum_{k=0}^{+\infty}(k+1)^2|f_k|^2. \tag{6.8}$$

We know that these mode sums have ultraviolet divergences, and that we must choose a particular regularization method to subtract them. In the particular cosmological model considered it is very helpful to carry out the regularisation of the energy-momentum tensor by adiabatic methods [7]. It is well known that to reproduce all the divergences of $\langle T_{\mu\nu}\rangle$ we need to calculate the adiabatic frequency to an order which includes all terms involving no more than four derivatives with respect to the comoving time variable t [3].

7. ADIABATIC EXPANSION AND REGULARIZATION

In this section we present the adiabatic expansion to calculate the adiabatic frequency up to second adiabatic order, that is, up to two derivatives (with respect to comoving time) in the metric tensor. In the most general case one needs to carry out the full fourth-order adiabatic subtraction. But in this case, and as we are considering a conformally coupled massive field, it is well known that the divergent fourth adiabatic order terms are proportional to $(6\xi - 1)$, and therefore vanish. There is a second reason to carry out the subtraction only to second order, and this has to do with the backreaction

[6] From now on the equation for the isotropic pressure is not given, as it sufficient to calculate the trace and the energy.

problem, which being out of the scope of this paper, will be discussed elsewhere [8]. The exact mode functions f_k satisfy equation (4.6),

$$\ddot{f}_k(t) + \Omega_k^2(t) f_k(t) = 0. \tag{7.1}$$

We can look for solutions of semiclassical nature (WKB)

$$\tilde{f}_k(t) = \frac{1}{[2W_k(t)]^{1/2}} \exp\left[-i \int_{t_0}^{t} dt' \; W_k(t') \right] \tag{7.2}$$

with $W_k(t)$ defined by [9]

$$W_k^2 = \Omega_k^2 + \frac{3}{4}\frac{\dot{W}_k^2}{W_k^2} - \frac{1}{2}\frac{\ddot{W}_k}{W_k} \qquad \text{with} \quad \Omega_k^2 = \frac{k^2}{C} + m^2 - \frac{1}{2}\left(\dot{H} + \frac{H^2}{2} \right). \tag{7.3}$$

In order to solve for $W_k(t)$ we need to carry out the following iteration:

$$W_k^{(0)2} \equiv \omega_k^2 = \frac{k^2}{\mathrm{C}} + m^2 \tag{7.4}$$

$$W_k^{(2)2} \equiv \Omega_k^2 + \frac{3}{4}\frac{\dot{W}_k^{(0)2}}{W_k^{(0)2}} - \frac{1}{2}\frac{\ddot{W}_k^{(0)}}{W_k^{(0)}} = \Omega_k^2 + \frac{3}{4}\frac{\dot{\omega}_k^2}{\omega_k^2} - \frac{1}{2}\frac{\dot{\omega}_k}{\omega_k} \tag{7.5}$$

$$W_k^{(4)2} \equiv \Omega_k^2 + \frac{3}{4}\frac{\dot{W}_k^{(2)2}}{W_k^{(2)2}} - \frac{1}{2}\frac{\dot{W}_k^{(2)}}{W_k^{(2)}}\,. \tag{7.6}$$

Let us consider the second adiabatic order mode functions, as for a conformally coupled massive field we do not need to go to adiabatic order four:

$$f_k^{(2)}(t) \stackrel{\text{def}}{=} \frac{1}{[2W_k^{(2)}(t)]^{1/2}} \exp\left[-i \int_{t_0}^{t} dt' \; W_k^{(2)}(t') \right]. \tag{7.7}$$

We need to calculate the energy, pressure, and trace for these modes, as these contributions will be the ones to be subtracted in the adiabatic regularization scheme. The quantities needed are

$$|f_k^{(2)}|^2 = \frac{1}{2W_k^{(2)}}, \qquad \Re[\dot{f}_k^{(2)} f_k^{(2)^*}] = -\frac{\dot{W}_k^{(2)}}{4W_k^{(2)2}}, \tag{7.8}$$

$$|\dot{f}_k^{(2)}|^2 = \frac{1}{2W_k^{(2)}}\left(W_k^{(2)2} + \frac{\dot{W}_k^{(2)2}}{4W_k^{(2)2}} \right)$$

Our goal is to obtain a renormalized and covariantly conserved expectation value for the energy-momentum tensor. By looking at the nature of the divergences that are contained in $\langle T_{\mu\nu}\rangle$ we conclude that they will be cancelled by performing a second-order adiabatic subtraction [10]. Furthermore, if we perform a higher order adiabatic subtraction, we do not obtain physical finite values for the energy, pressure, and trace. How do we know? We can compare [11] with other regularization techniques, such as covariant point splitting [12], that yield a finite and covariantly conserved $\langle T_{\mu\nu}\rangle$.

8. PARTICLE CREATION IN A DE SITTER UNIVERSE

8.1 Discussion of the Choice of the Generalized Bogoliubov Transformation

The symmetry of the FRW metric[7] makes the field equations separable, so that the modes satisfy a time-dependent harmonic oscillator type of differential equation. These equations have analytic solutions in very specific cosmological backgrounds, and only for particular values of the coupling between the quantum and the gravitational fields. It is therefore necessary to develop a numerical scheme that can solve these equations for a wider range of situations. However, there are some technical obstacles that have to be overcome prior to carrying out any numerical simulation. The first is the presence of divergences when calculating the expectation value of the bare energy-momentum tensor in the vacuum state chosen. We need to regularize and renormalize this expectation value to obtain the physical values for the energy, trace, and pressure of the quantum field. The second is the fact that in order to do a backreaction analysis the equations become fourth order in time derivatives of the classical metric and a special scheme has to be developed in order to solve them [14].

We have recently found that in order to isolate the divergences of the energy, trace, and pressure, and to be able to have a physical and intuitive understanding of particle creation, it is necessary to make a generalized Bogoliubov transformation that relates the exact mode functions to the second adiabatic order mode functions [6]. Such a transformation possesses the following advantages:

1. The divergent structure of the energy, trace, and pressure becomes transparent, and it is therefore natural to choose adiabatic subtraction as our regularization scheme. Not only do we only need to go to adiabatic order two (instead of four, as the fourth-order adiabatic divergences are purely logarithmic), but the subtraction can be per-

[7] A de Sitter cosmological model is a particular type of FRW universe.

formed before doing the numerical calculation, which is a great advantage when carrying out the simulation.

2. With the new scheme the renormalized energy, trace, and pressure satisfy the covariant conservation of the energy-momentum tensor; the trace anomaly is recovered; and the physical parameters (energy, trace, and pressure) have a very intuitive and physical interpretation in terms of the number of particles created $\mathcal{N}_k$ and its first and second time derivatives.
3. In four dimensions the expectation value of the energy-momentum tensor has quartic, quadratic, and logarithmic divergences. With our present scheme, it is not necessary to subtract the fourth adiabatic order logarithmic divergences. They can be scaled away in a numerical calculation by a proper rescaling of the cutoff used in the mode sums. This technical issue is a great simplification when carrying out the numerical simulation.

Some other physical arguments that support our approach are:

4. In the most general case,[8] it is well known that all the divergences in $\langle T_{\mu\nu} \rangle$ will be canceled by performing an adiabatic subtraction of order four [3]. In a de Sitter universe and for $\xi = 1/6$ the only divergences appearing are of adiabatic order zero. We still need to go to one adiabatic order higher, as there are finite pieces to be subtracted that are adiabatic order two. These terms are crucial to get the proper trace anomaly in the massless case.
5. If we perform a higher order adiabatic subtraction we do not obtain the physical finite values for the energy, pressure, and trace. We want to obtain the proper trace anomaly. It is at this stage that the finite pieces become important, as one does not obtain the physical, renormalized, and covariantly conserved $\langle T_{\mu\nu} \rangle_R$ if one does not subtract all the finite terms needed.
6. How do we know that we are going to obtain a covariantly conserved energy-momentum tensor? In order to assure ourselves, we only need to compare with other covariant regularization schemes, such as, for example covariant point splitting [12], that yield a finite and covariantly conserved $\langle T_{\mu\nu} \rangle$. In particular, by choosing a point splitting in the space direction, we can compare with adiabatic subtraction by carrying out the mode integrals with a momentum cutoff.

[8] We are still limited to a FRW cosmological model, but the coupling constant ξ can take any value.

8.2. Bogoliubov Transformation and Initial Conditions

We start with the exact mode functions f_k. We define the generalized Bogoliubov transformation in terms of the instantaneous adiabatic basis (see section 5).

$$f_k(t) \stackrel{\text{def}}{=} \alpha_k(t)\,\tilde{f}_k(t) + \beta_k(t)\tilde{f}_k^*(t) \tag{8.1}$$

with α_k and β_k time-dependent functions. We know that this condition does not uniquely determine α_k and β_k, and that we have to give an extra condition (that is consistent with the Wronskian) on $\dot{f}_k$,

$$\dot{f}_k(t) = \left[-iW_k(t) + \frac{V_k(t)}{2}\right]\alpha_k(t)\,\tilde{f}_k(t) + \left[iW_k(t) + \frac{V_k(t)}{2}\right]\beta_k(t)\,\tilde{f}_k^*(t) \tag{8.2}$$

with

$$\tilde{f}_k(t) = \frac{1}{[2W_k(t)]^{1/2}}\exp\left[-i\int_{t_0}^{t} dt'\; W_k(t')\right],$$

$$S_k(t) \stackrel{\text{def}}{=} \int_{t_0}^{t} dt'\; W_k(t') \tag{8.3}$$

$$W_k^2 \equiv W_k^{(2)2} = \omega_k^2 + A_k,$$

$$A_k = -\frac{m^2}{2\omega_k^2}(\dot{H} + 3H^2) + \frac{5m^4H^2}{4\omega_k^4}, \tag{8.4}$$

$$V_k \stackrel{\text{def}}{=} -\frac{\dot{\omega}_k}{\omega_k} = H\left(1 - \frac{m^2}{\omega_k^2}\right).$$

These equations determine α_k and β_k uniquely,

$$\alpha_k(t) = i\tilde{f}_k^*(t)\left[\dot{f}_k(t) - \left(iW_k(t) + \frac{V_k(t)}{2}\right)f_k(t)\right]$$

$$\beta_k(t) = -i\tilde{f}_k(t)\left[\dot{f}_k(t) + \left(iW_k(t) - \frac{V_k(t)}{2}\right)f_k(t)\right]. \tag{8.5}$$

Define also the following variables (*adiabatic basis variables*)[9]

[9] $\Re(z)$ and $\Im(z)$ mean, respectively, the real and imaginary parts of the complex number z.

$$\mathcal{N}_k(t) \stackrel{\text{def}}{=} \beta_k(t)\beta_k^*(t) \tag{8.6}$$

$$\mathcal{C}_k^R(t) \stackrel{\text{def}}{=} \Re[\alpha_k(t)\beta_k^*(t)\exp(-2iS_k(t))] \tag{8.7}$$

$$\mathcal{C}_k^I(t) \stackrel{\text{def}}{=} \Im[\alpha_k(t)\beta_k^*(t)\exp(-2iS_k(t))] \tag{8.8}$$

which can be used to write

$$|f_k(t)|^2 = \frac{1 + 2\mathcal{N}_k(t)}{2W_k(t)} + \frac{\mathcal{C}_k^R(t)}{W_k(t)} \tag{8.9}$$

$$\begin{aligned} |\dot{f}_k(t)|^2 &= \frac{1 + 2\mathcal{N}_k(t)}{2W_k(t)}\left(W_k^2(t) + \frac{V_k^2(t)}{4}\right) \\ &\quad + \frac{\mathcal{C}_k^R(t)}{W_k(t)}\left(-W_k^2(t) + \frac{V_k^2(t)}{4}\right) + \mathcal{C}_k^I(t)V_k(t) \end{aligned} \tag{8.10}$$

$$\Re[f_k^*(t)\dot{f}_k(t)] = \frac{1 + 2\mathcal{N}_k(t)}{4W_k(t)}V_k(t) + \frac{\mathcal{C}_k^R(t)}{2W_k(t)}V_k(t) + \mathcal{C}_k^I(t). \tag{8.11}$$

We first have to choose the initial conditions. We choose adiabatic initial conditions as described in Section 5:

$$f_k(t_0) = [2W_k(t_0)]^{-1/2}, \qquad \dot{f}_k(t_0) = \left[-iW_k(t_0) + \frac{V_k(t_0)}{2}\right]f_k(t_0) \tag{8.12}$$

so that at t_0 we have

$$\alpha_k(t_0) = 1, \qquad \beta_k(t_0) = 0 \tag{8.13}$$

therefore the labeling of adiabatic initial conditions.

We write now the equations for the bare energy and trace in terms of the previously introduced adiabatic basis variables $\mathcal{N}_k$, $\mathcal{C}_k^R$, and $\mathcal{C}_k^I$:

$$\langle\epsilon\rangle = \frac{1}{4\pi^2a^3}\sum_{k=0}^{+\infty}(k+1)^2[\epsilon_k^{\mathcal{N}}(\mathcal{N}_k + 1/2) + \epsilon_k^R\mathcal{C}_k^R + \epsilon_k^I\mathcal{C}_k^I] \tag{8.14}$$

$$\langle T\rangle = \frac{1}{4\pi^2a^3}\sum_{k=0}^{+\infty}(k+1)^2[T_k^{\mathcal{N}}(\mathcal{N}_k + 1/2) + T_k^R\mathcal{C}_k^R + T_k^I\mathcal{C}_k^I \tag{8.15}$$

where

$$\epsilon_k^{\mathcal{N}} = \frac{1}{W_k}\left(W_k^2 + \frac{V_k^2}{4} - \frac{HV_k}{2} + \omega_k^2 + \frac{H^2}{4}\right) \tag{8.16}$$

$$\epsilon_k^R = \frac{1}{w_k}\left(-W_k^2 + \frac{V_k^2}{4} - \frac{HV_k}{2} + \omega_k^2 + \frac{H^2}{4}\right) \tag{8.17}$$

$$\epsilon_k^I = V_k - H \tag{8.18}$$

$$T_k^{\mathcal{N}} = -\frac{2m^2}{W_k}, \qquad T_k^R = -\frac{2m^2}{W_k}, \qquad T_k^I = 0. \tag{8.19}$$

8.3. Regularization and Renormalization

In the previous section we have been able to write the energy and trace in terms of the instantaneous adiabatic basis variables in such a way that the divergences are isolated and appear in the true vacuum contribution (vacuum polarization) to the energy-momentum tensor, and the term in $\langle T_{\mu\nu}\rangle$ coming from particle creation is finite.

The energy and trace of the vacuum as "measured" by the instantaneous adiabatic observers are, respectively, given by

$$\langle\epsilon\rangle_{\text{vac}} \stackrel{\text{def}}{=} \frac{1}{4\pi^2 a^3} \sum_{k=0}^{+\infty} (k+1)^2 \frac{\epsilon_k^{\mathcal{N}}}{2} \tag{8.20}$$

$$\langle T\rangle_{\text{vac}} \stackrel{\text{def}}{=} \frac{1}{4\pi^2 a^3} \sum_{k=0}^{+\infty} (k+1)^2 \frac{T_k^{\mathcal{N}}}{2} \tag{8.21}$$

and the energy and trace of the created particles $\langle\epsilon\rangle_{\text{matter}}$ are, respectively, given by

$$\langle\epsilon\rangle_{\text{matter}} \stackrel{\text{def}}{=} \langle\epsilon\rangle - \langle\epsilon\rangle_{\text{vac}} = \frac{1}{4\pi^2 a^3} \sum_{k=0}^{+\infty} (k+1)^2 (\epsilon_k^{\mathcal{N}} \mathcal{N}_k + \epsilon_k^R \mathcal{C}_k^R + \epsilon_k^I \mathcal{C}_k^I) \tag{8.22}$$

$$\langle T\rangle_{\text{matter}} \stackrel{\text{def}}{=} \langle T\rangle - \langle T\rangle_{\text{vac}} = \frac{1}{4\pi^2 a^3} \sum_{k=0}^{+\infty} (k+1)^2 (T_k^{\mathcal{N}} \mathcal{N}_k + T_k^R \mathcal{C}_k^R + T_k^I \mathcal{C}_k^I) \tag{8.23}$$

It would be desirable to know that all the divergences are buried in the vacuum piece, so that we have a finite term that can be interpreted as only due to particle creation. This is where the new approach presented here brings good news, as it can be shown that all the divergences are contained in the vacuum contribution [13] (in fact, as this is a new approach, it has to be explicitly shown). We subtract up to adiabatic order two to obtain the renormalized energy-momentum tensor. For closed cosmological models the appropriate adiabatic subtraction is the one that has continuous measure [10]. This means the following replacement:

$$\sum_{k=0}^{+\infty} (k+1)^2 \Rightarrow \int_0^{+\infty} dk\, k^2.$$

We define the renormalized energy and trace of the vacuum as follows:

$$\langle \epsilon \rangle_{\rm vac}^{R} \stackrel{\rm def}{=} \langle \epsilon \rangle_{\rm vac} - \langle \epsilon \rangle_{\rm vac}^{\rm A2} \tag{8.24}$$

$$\langle T \rangle_{\rm vac}^{R} \stackrel{\rm def}{=} \langle T \rangle_{\rm vac} - \langle T \rangle_{\rm vac}^{\rm A2} \tag{8.25}$$

where the second-order adiabatic energy and trace are given by

$$\begin{aligned}\langle \epsilon \rangle_{\rm vac}^{A2} &= \frac{1}{4\pi^2 a^3}\int_0^{+\infty} dk\, k^2\left(\omega_k + \frac{H^2 m^4}{8\omega_k^5}\right)\\ &= \frac{1}{4\pi^2 a^3}\sum_{k=0}^{+\infty}(k+1)^2\left(\omega_k + \frac{H^2 m^4}{8\omega_k^5}\right) - \langle \epsilon \rangle_{\rm vac}^{\rm Plana}\end{aligned} \tag{8.26}$$

$$\begin{aligned}\langle T \rangle_{\rm vac}^{A2} &= -\frac{1}{4\pi^2 a^3}\int_0^{+\infty} dk\, k^2\left[\frac{m^2}{\omega_k} + \frac{m^4}{4\omega_k^5}(\dot{H} + 3H^2) - \frac{5m^6 H^2}{8\omega_k^7}\right]\\ &= -\frac{1}{4\pi^2 a^3}\sum_{k=0}^{+\infty}(k+1)^2\left[\frac{m^2}{\omega_k} + \frac{m^4}{4\omega_k^5}(\dot{H} + 3H^2) - \frac{5m^6 H^2}{8\omega_k^7}\right] - \langle T \rangle_{\rm vac}^{\rm Plana}\end{aligned} \tag{8.27}$$

with the Plana terms being a finite contribution [10]. It can be shown (by carrying out the adiabatic expansion of W_k; see Section 7) that as far as the divergent terms are concerned

$$\text{Divergences of } (\langle T_{\mu\nu} \rangle_{\rm vac}) = \text{Divergences of } (\langle T_{\mu\nu} \rangle_{\rm vac}^{\rm A2}) \tag{8.28}$$

We define the renormalized energy-momentum tensor as

$$\langle T_{\mu\nu} \rangle_R \stackrel{\rm def}{=} \langle T_{\mu\nu} \rangle - \langle T_{\mu\nu} \rangle_{\rm vac}^{\rm A2} = \langle T_{\mu\nu} \rangle_{\rm vac} + \langle T_{\mu\nu} \rangle_{\rm matter} - \langle T_{\mu\nu} \rangle_{\rm vac}^{\rm A2} \tag{8.29}$$

9. NUMERICAL RESULTS

In this section we present numerical results for different finite-time initial conditions. In all four cases considered the parameters are $m = 0.6$, $\xi = 1/6$ and we have studied the evolution of 100 modes.

Before presenting the results it is interesting to describe the physics of the process taking place. We study the evolution of the mode $f_k(t)$ as a function of time ($k \equiv k_{\rm com}$ is the comoving momentum). The physical momentum associated with this mode is $k_{\rm phy} = k_{\rm com}/a(t)$. For a de Sitter universe the characteristic length scale is the inverse of the Hubble time function $H^{-1} = a/\dot{a}$. It is therefore reasonable to expect that particle production will occur when horizon crossing takes place. By this we mean the following:

$$k_{\rm phy} \propto H(t) \Rightarrow k_{\rm phy} \propto \frac{\dot{a}(t)}{a(t)}. \tag{9.1}$$

Let us denote by t_k the time at which the mode $f_k(t)$ crosses the horizon. The equation defining t_k is

$$k_{\rm com} \propto \dot{a}(t_k). \tag{9.2}$$

In this representation of the de Sitter universe the scale factor is $a(t) = Z^{-1} \cosh(Zt)$ and it can be easily seen that there are two times of crossing for each mode f_k. The first one takes place at $-t_k$, when the mode is entering the horizon, and the second one at $+t_k$, when the mode is leaving the horizon. In this sense, we can expect two resonances for the number of particles created in the *kth* mode at $\pm\ t_k$. We write then $k_{\rm com} = \aleph \dot{a}(t_k)$ with $\aleph$ a proportionality constant that will be determined from the numerical simulations. We also expect to have a constant particle production rate when the mode is inside the horizon, that is, between $-t_k$ and t_k. To summarize the previous comments: at the initial time t_0 the mode f_k is such that $\beta_k(t_0) = 0$, so that no particles are present; particle production in the *kth* mode will be relevant at the horizon crossing times ($\pm t_k$); when the mode is inside the horizon, particle production is frozen; once the mode exits the horizon, it freezes again, contributing to a constant rate of particle production.

It is important to make a comment about the late-time behavior of the system. We know that for any time t

$$f_k(t) = \alpha_k(t)\tilde{f}_k(t) + \beta_k(t)\tilde{f}_k^*(t). \tag{9.3}$$

We also know that there are exact solutions $f_{k(+)}$ such that at late times they have adiabatic behavior

$$\lim_{t\to\pm\infty} f_{k(\pm)}(t) = \tilde{f}_k(t). \tag{9.4}$$

Both f_k and $f_{k(+)}$ are exact solutions of the mode equation (4.6), and therefore they must be related by a time-independent Bogoliubov transformation

$$f_k(t) = A_k f_{k(+)}(t) + B_k \tilde{f}_{k(+)}^*(t). \tag{9.5}$$

It is then easy to see that the later-time behavior of α_k and β_k is determined by the coefficients A_k and B_k:

$$\lim_{t\to+\infty} \alpha_k(t) = A_k, \qquad \lim_{t\to+\infty} \beta_k(t) = B_k. \tag{9.6}$$

We call this regime the asymptotic late-time regime, and it corresponds to the freezeout of the modes once they leave the horizon at time $+t_k$. The following three situations can arise:

1. $t_0 < -t_k$. In this case the mode will enter the horizon at $-t_k$, then freeze inside the horizon, and finally exit at time t_k, to reach its asymptotic late-time regime. The mode has two resonances at $\pm t_k$.
2. $-t_k < t_0 < t_k$. In this case the mode is born inside the horizon, and it will exit at time t_k, to reach its asymptotic late-time regime. The mode has one resonance at t_k.
3. $t_0 > t_k$. The mode is born outside the horizon, and will never get inside it. There is no resonance in this case, and the particle number has a smooth transition from zero to its asymptotic late regime.

9.1. Initial Time $t_0 = -8$

This initial condition corresponds to the first case for all the modes considered in the simulation. The two horizon crossings can be seen in Fig. 1. The middle plateau corresponds to the time the modes spend inside the horizon, and the final plateau corresponds to the late-time asymptotic regime.

Figure 2 shows the cutoff dependence of $\langle \epsilon \rangle_{\text{matter}}$. This is so because the number of modes considered is not high enough. Notice also that the higher the k, the earlier the first horizon crossing takes place, and there are many more modes that contribute significantly and have not been included in the simulation.

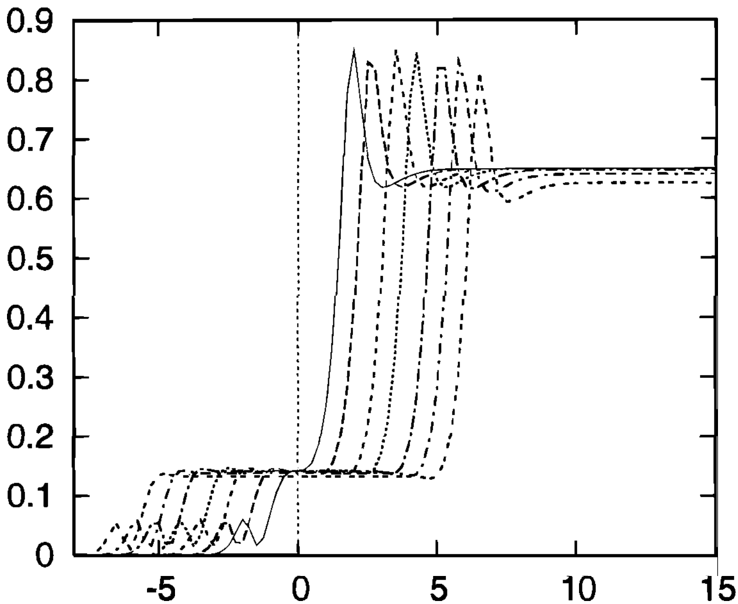

Fig. 1. Time evolution of the particle number N_k for $k = 0, 1, 4, 9, 19, 49,$ and 99.

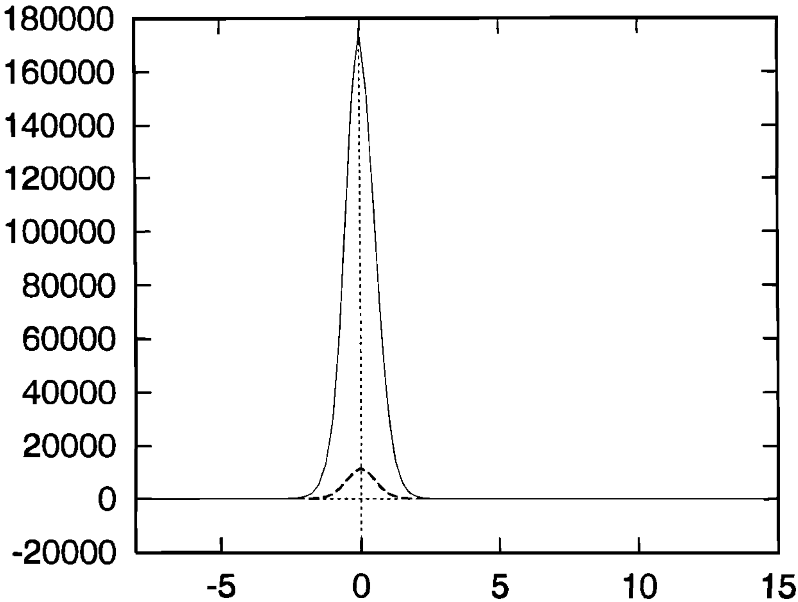

Fig. 2. Time evolution of the energy of the created particles for $k = 49$ and 99.

9.2. Initial Time $t_0 = -3$

The zero mode presents two resonances. Modes 1 and 4 are born after their first horizon crossing could have taken place. The rest of the modes are born at times such that they are still not excited when they are inside the horizon (there is no middle plateau for them; see Fig. 3), and only show one resonance upon exiting.

Figure 4 shows a small cutoff dependence of $\langle T \rangle_{\text{matter}}$. This is so because the number of modes considered is high enough to take into account all those that contribute.

9.3. Initial Time $t_0 = 0$

In Fig. 5 it can be seen that this initial condition corresponds to one in which all the modes considered show one resonance only. They are born inside the horizon and therefore can only exit it at time t_k.

In Fig. 6 we can see that for times before 4 we have considered enough modes to yield the full finite contribution to the trace (dotted), pressure (dashed), and energy (solid) of the created particles. At late times more modes should be taken into account.

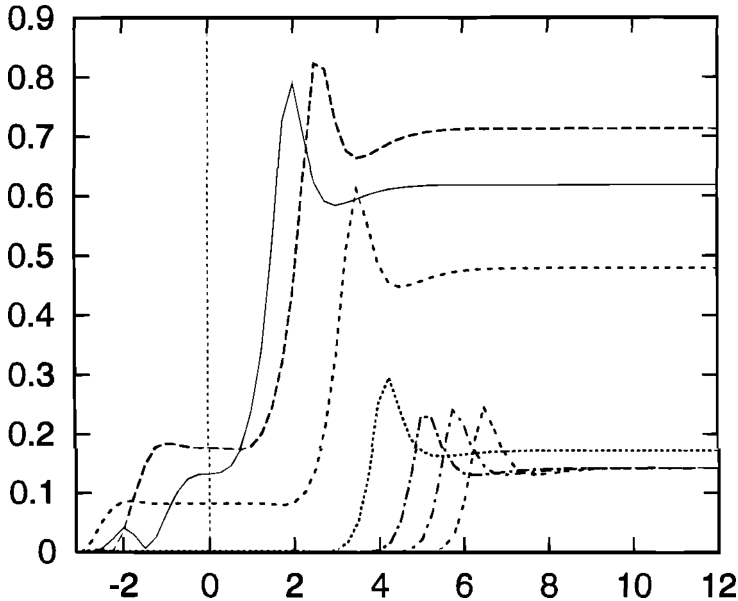

Fig. 3. Time evolution of the particle number N_k for $k = 0, 1, 4, 9, 19, 49$, and 99.

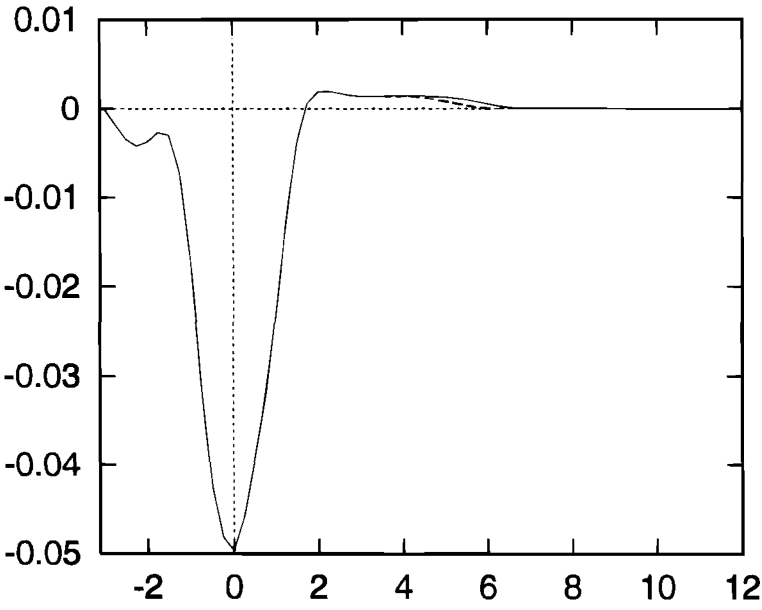

Fig. 4. Time evolution of the trace of the created particles for $k = 49$ and 99.

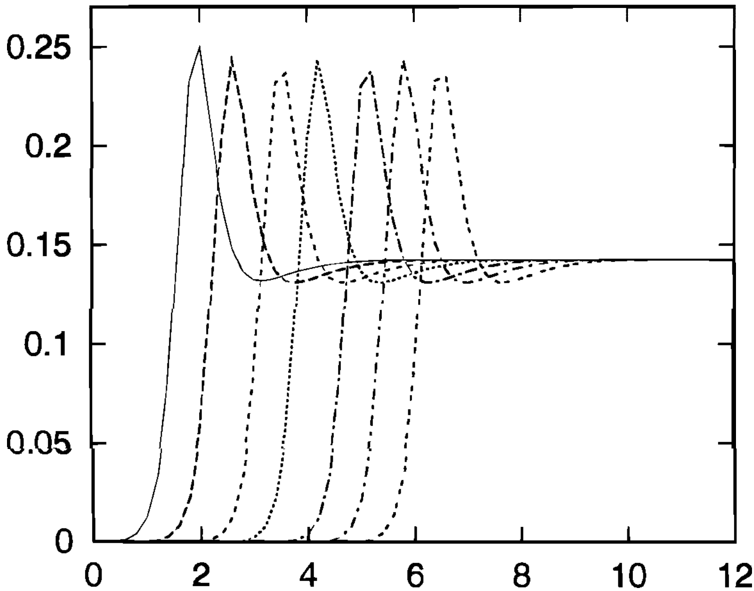

Fig. 5. Time evolution of the particle number for N_k for k = 0, 1, 4, 19, 49, and 99.

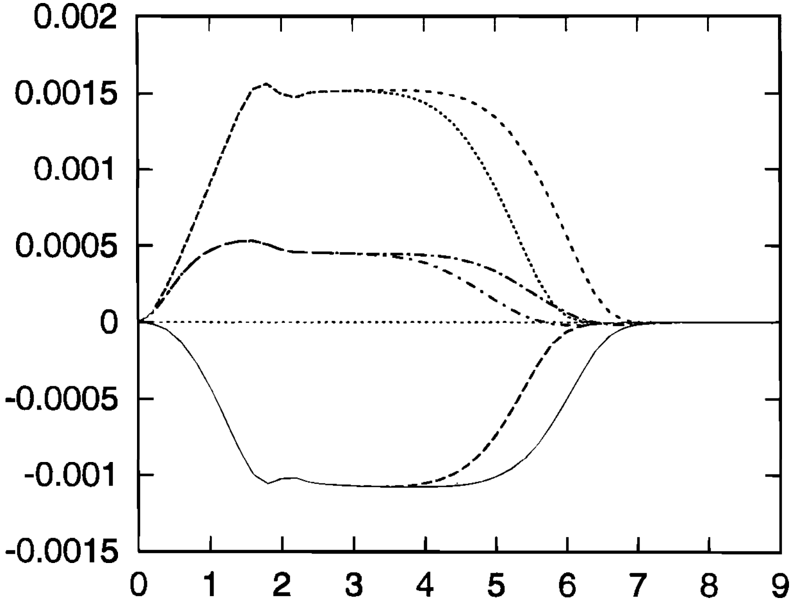

Fig. 6. Time evolution of the trace, pressure, and energy of the created particles for k = 49 and 99.

9.4. Initial Time $t_0 = 3$

In Fig. 7 it can be seen that modes 0 and 1 are born outside the horizon, and in their late-time regime. Mode 4 is born almost at the same time as it is exiting the horizon. The rest of the modes are born inside the horizon and therefore they can only exit it at time t_k.

In Fig. 8 we can see that for times before 4 we have considered enough modes to yield the full finite contribution to the trace (dotted), pressure (dashed), and energy (solid) of the created particles. At late times more modes should be taken into account. This initial condition is very similar to the previous one considered, $t_0 = 0$.

10. CONCLUSIONS AND WORK IN PROGRESS

1. It is essential to choose appropriate initial conditions, as these determine the initial state of the quantum system.
2. If one is studying particle production in the early universe, it is essential to consider non-conformally invariant fields.
3. We have introduced an adiabatic particle basis in which the energy-momentum tensor has a very intuitive and physical meaning. The divergent terms correspond to vacuum polarization, and the finite terms are particle production contributions.

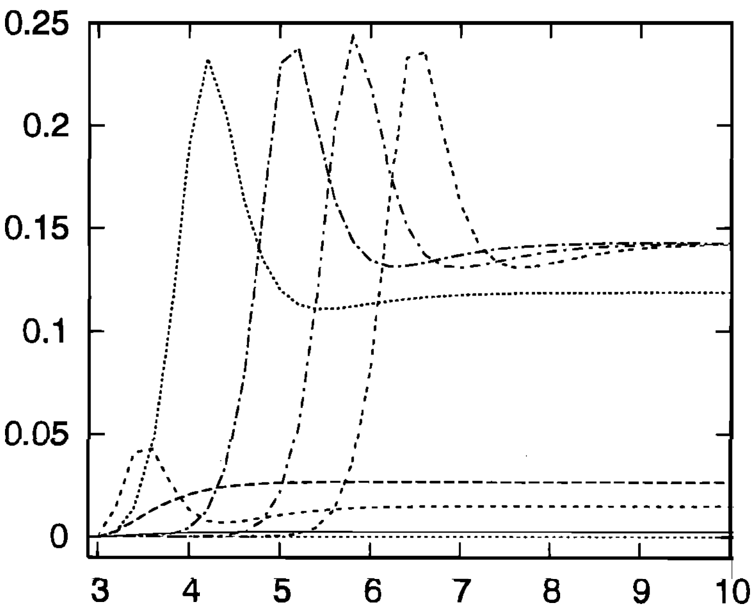

Fig. 7. Time evolution of the particle number N_k for $k = 0, 1, 4, 9, 19, 49$, and 99.

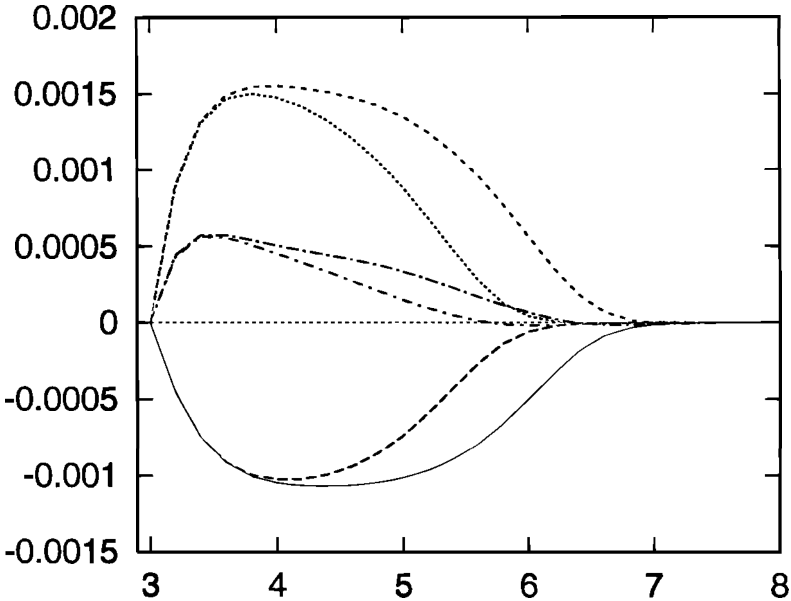

Fig. 8. Time evolution of the trace, pressure, and energy of the created particles for $k =$ 49 and 99.

4. We have regularized $\langle T_{\mu\nu} \rangle$ by performing a second-order adiabatic subtraction.
5. The particular choice of $\xi = 1/6$ can be generalized, with no essential changes in the formalism.
6. The numerical results presented display the nature of the particle production effect, showing that it becomes relevant whenever there is horizon crossing. All three possible scenarios (before crossing, inside, and after crossing the horizon) have been considered.
7. At the moment we have some hints that the backreaction (of the quantum matter fields on the classical gravitational background) will be important, as the values of the energy, pressure, and trace are not red-shifted away (due to the expansion of the cosmological model) in the time evolution of the system (see Figs. 6 and 8).
8. The formalism presented here can be applied for arbitrary ξ, m, and $a(t)$. For example, when $m = 0 = \xi$ we could treat a linearized quantum gravitational field.
9. The study of interacting field theories is also part of the future application of this formalism.
10. A proper study of backreaction should include the dynamics of the scale factor $a(t)$.

ACKNOWLEDGMENTS

The author wants to express her gratitude to the organizers of the workshop on Quantum Gravity in the Southern Cone for their hospitality and consideration. This work was supported in part by the U.S. Department of Energy and the U.S. National Science Foundation. I wish to thank Salman Habib, Emil Mottola, Stephen Ramsey and Joachim Lindig for many useful discussions, for their inspiration and encouragement, and for a careful reading of the manuscript.

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